Tangent Line Problem/Derivative of a Function

$$m_{sec} = \frac{f(c+\Delta x) - f(c)}{(c+\Delta x) - c}$$

$$= \frac{f(c+\Delta x)-f(c)}{\Delta x}$$ = **Difference Quotient**

<u>Defn:</u> If f is defined on an open interval containing c, and if the limit

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x)-f(c)}{\Delta x} = m$$

exists, then the line passing through *(c, f(c))* with slope m is the <u>tangent line</u> to the graph of f at the point *(c, f(c)).* (m is called <u>slope of the graph of f at x=c</u>)

<u>EX</u>: Find the slope of the graph $f(x) = x^2 - 2x - 3$ at the point (3, 0)

Defn: The <u>derivative</u> of f at x is given by:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

provided the limit exists.

<u>Terms</u>:

<u>Differentiation</u>: process of finding the derivative of a function

<u>Differentiable</u>: if a functions derivative exists at x

<u>Notation</u>: $f'(x)$, $\frac{dy}{dx}$, y', $\frac{d}{dx}[f(x)]$, $D_x[y]$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$= f'(x)$$

<u>EX</u>: Find the derivative of $f(x) = x^3 + x^2$

Note: If f is continuous at c and $\lim_{\Delta x \to 0} \left| \frac{f(c+\Delta x)-f(c)}{\Delta x} \right| = \infty$ then the vertical line, $x = c$, passing through *(c, f(c))* is a vertical tangent line.

EX: Find an *equation of the tangent line* to the graph of f at the given point.

$$f(x) = x^2 + 2x + 1, \ (-3, 4)$$

Alternate Limit Form : $f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$ *Provided this limit EXISTS!!!!*

i.e., $\lim_{x \to c^-} \frac{f(x)-f(c)}{x-c} = \lim_{x \to c^+} \frac{f(x)-f(c)}{x-c}$

Note: If a function is *not* continuous at $x = c$ then it is also *not* differentiable at $x = c$.

Theorem: If f is differentiable at $x=c$, then f is continuous at $x=c$.

EX: $f(x) = [\![x]\!]$

******Functions which are discontinuous at *x*=*c*, have a corner at *x*=*c*, or that have a vertical tangent at *x*=*c* are NOT DIFFERENTIABLE!!!!!!!!!!!!!!**

Investigate:

1) $f(x) = x^{\frac{2}{3}} + 1$
2) $f(x) = x^{\frac{1}{3}}$
3) $f(x) = |x + 3|$